2009
MATHCOUNTS STATE COMPETITION

SPRINT ROUND

1. There are 24 books on 3 shelves. The top shelf has 8 mysteries, the middle has 10 math books and the bottom has 6 science books.
We take 2 mysteries, 2 math books and 2 science books off the shelf. That leaves 6 mysteries, 8 math books and 4 science books. So what fraction are math books?
We now have a total of 24 – 6 = 18 books and we have 8 math books.
$\frac{8}{18} = \frac{4}{9}$ **Ans.**

2. There are 12 eggs in each egg carton. So what is the smallest number of eggs > 10,000 that will fill an integer number of cartons.
$\frac{10{,}000}{12}$ = 833 R4
This means that the last carton has only 4 eggs in it. To fill the whole carton we will need 12 – 4 = 8 more eggs.
10,000 + 8 = 10,008 **Ans.**

3. Wilhelm has seven tokens with a number written on each token's top face and the numbers are distinct consecutive primes. So what's the least possible sum of the prime numbers?
List the 7 smallest primes.
2, 3, 5, 7, 11, 13, 17
2 + 3 + 5 + 7 + 11 + 13 + 17 = 58 **Ans.**

4. A ψ B = 2A + 5B
3 ψ 1 = (2 × 3) + (5 × 1) = 6 + 5 = 11
9 ψ 11 = (2 × 9) + (5 × 11) = 18 + 55 = 73 **Ans.**

5. A flock of geese loses $\frac{2}{5}$ of their members when a motorcycle drives past. Then, it loses $\frac{1}{3}$ of their remaining members when a herd of deer runs past. Finally, another 24 geese take off after a loud siren sounds leaving only 46 geese. So how many geese were there originally?
Let x = the number of geese
After the motorcycle goes by we have
$x - \frac{2}{5}x = \frac{3}{5}x$ geese left.
Then, after the deer there are
$\frac{3}{5}x - \left(\frac{1}{3}\right)\left(\frac{3}{5}x\right) = \frac{2}{5}x$ geese left.
And we know that
$\frac{2}{5}x - 24 = 46$
$\frac{2}{5}x = 46 + 24 = 70$
$x = \frac{70 \times 5}{2}$ = 35 × 5 = 175 **Ans.**

6. The probability that it will rain on Saturday is 60%. The probability that it will rain on Sunday is 25%. So the probability that it will rain on Saturday **and** Sunday is:
$\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$ = 15% **Ans.**

7. We have 5 termites eating wood.
Woody is 20mm ahead of Muncher.
Cruncher is 10 mm behind Woody.
Muncher is 5 mm behind Nibbler.
Biter is 15 mm ahead of Cruncher.
(Who made up these names????)
And we are asked to find the distance between the two termites that are the farthest apart.
Let W = Woody
Let C = Cruncher
Let M = Muncher
Let N = Nibbler
Let B = Biter
Get 4 of the termites' positions with respect to the fifth termite.
W = M + 20
C = W – 10
M = N – 5
B = C + 15
W = (N – 5) + 20 = N + 15
C = (N + 15) – 10 = N + 5
B = (N + 5) + 15 = N + 20
So what do we have?
N – 5, N + 15, N + 5, N + 20 and N.
The termite furthest back is N – 5. The termite farthest ahead is N + 20.
20 – (–5) = 20 + 5 = 25 **Ans.**

8. Four primes, *a*, *b*, *c* and *d* form an increasing arithmetic sequence.

a > 5 and the common difference is 6.
So what is the ones digit of a?
If *a* > 5, what is the ones digit of *a*?
Suppose *a* is 7. Then the 4 primes would be 7, 13, 19, 25. No, 25 is not a prime.
Suppose *a* is 11. Then the 4 primes would be 11, 17, 23, 29. That's just fine.
1 **Ans.**

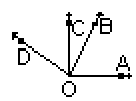
9. 7 ligs = 4 lags
9 lags = 20 lugs
How many ligs are equivalent to 80 lugs?
Let *i* = ligs
Let *a* = lags
Let *u* = lugs
$7i = 4a$
$9a = 20u$
The LCM of 4 and 9 is 36.
$63i = 36a$
$36a = 80u$
$63i = 80u$
63 **Ans.**

10. When a tank is $\frac{1}{6}$ full, 130 gallons of water are added and this makes the tank $\frac{3}{5}$ full. So how many gallons does the tank hold?
130 gallons is $\frac{3}{5} - \frac{1}{6} = \frac{18}{30} - \frac{5}{30} = \frac{13}{30}$ of the tank.
Therefore, 10 gallons is $\frac{1}{30}$ of the tank.
30 × 10 = 300 **Ans.**

11. 30 students took a test. You could get a score of 3 to 10 on this test. Exactly 24 students earned a score of 7 or higher. We are asked to find the highest possible average of the 30 scores?
Since we want to find the highest possible average, let's suppose that the 24 students who earned a score of 7 or higher each got a 10. And let's also suppose that the other 6 students "almost" passed, i.e., they got a score of 6.
Then (24 × 10) + (6 × 6) =
240 + 36 = 276
This is the sum of their scores. We'll divide it by 30 to get the average.
$\frac{276}{30} = \frac{92}{10}$ = 9.2 **Ans.**

12. The mean of 5 integers is 1.5 times the median. Four of the integers are 8, 18, 36 and 62. The largest integer is not 62 (ah, that must mean something!!!) So what is the largest integer?
If 62 isn't the largest integer, then *x* must be the largest integer because all the other integers are less than 62. The median must obviously be 36.
1.5 times the median is 36 + 18 = 54
$\frac{8 + 18 + 36 + 62 + x}{5} = 54$
$\frac{124 + x}{5} = 54$
$124 + x = 270$
$x = 270 - 124 = 146$ **Ans.**

13. We are given the following information. OA is perpendicular to OC and OB is perpendicular to OD. *m*<AOD is 3.5 times *m*<BOC. We are asked to find *m*<AOD.



Let *x* = *m*<AOB
Let *y* = *m*<BOC
Let *z* = *m*<COD
Let *w* = *m<AOD*
Since OA is perpendicular to OC,
$x + y = 90$
Since OB is perpendicular to OD,
$z + y = 90$
This means that $x = z$
Also $w = x + y + z = 3.5y$
$x + z = 2.5y$
Since $x = z$,
$2x = 2.5y$
$x = 1.25y$
$1.25y + y = 90$
$2.25y = 90$
$y = 40$
Therefore,
$w = 3.5y = 40 \times 3.5 = 140$ **Ans.**

14. Three standard 6-faced dice are rolled. So what is the probability that the sum of the 3 numbers rolled is 9? How can

we get 9?

1, 2, 6	2, 2, 5	3, 3, 3
1, 3, 5	2, 3, 4	
1, 4, 4		

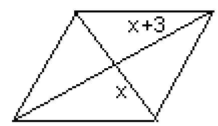
When all 3 numbers are different there are 6 ways to get that combination. So 1, 2, 6 and 1, 3, 5 and 2, 3, 4 each have 6 combinations. That's 18.
When there are only 2 different numbers there are 3 combinations. So 1, 4, 4 and 2, 2, 5 each have 3 combinations for a total of 6 combinations.
When all numbers are the same there is only 1 combination. 3, 3, 3 has 1 combination.
18 + 6 + 1 = 25 combinations.
The total number of combinations is
6 × 6 × 6 = 216
$\frac{25}{216}$ **Ans.**

15. Alfred, Brandon and Charles run a race. In how many different ways can the 3 finish if it is possible for 2 or more participants to finish in a tie?
Let's start without a tie. Then the number of ways is 3! = 6.
Now let's do the ties. If all 3 tie, then there's only 1 way. If only 2 tie, then we have 3 ways (Alfred and Brandon tie, Alfred and Charles tie and Brandon and Charles tie.) when 2 people tie for first and 3 ways when 2 people tie for last.
6 + 1 + 3 + 3 = 13 **Ans.**

16. A rectangular garden is such that $l = 2w$ where l is the length and w is the width. The perimeter is doubled and the new shape is a square with area 3600 sq. feet. So what is the area of the original garden?
If the new garden is a square with 3600 sq. feet, then each side of the square is 60 feet. The perimeter of the new garden is 240 feet. The perimeter of the old garden was $2(l + w) = 2(2w + w) = 6w$. This was doubled to $12w$ or 240.
$12w = 240$
$w = 20$
$l = 2w = 40$
The area of the old garden was
20 × 40 = 800 **Ans.**

17. What is the ordered pair (x, y) where x and y are integers and $2^x - 2^y = 8$
So what power of 2 subtracted from another power of 2 gives 8. Let's look at the powers of 2.
2, 4, 8, 16, 32, 64, 128 etc.
By inspection, we have to take the next power of 2 larger than 8 and then subtract 8 from it. I.e.,
16 – 8 = 8
Therefore, $x = 4$ and $y = 3$.
(4, 3) **Ans.**

18. The diagonals of a rhombus are always perpendicular bisectors of each other. We have a rhombus with side length $\sqrt{89}$ and diagonals that differ by 6 units. Let 2x be one diagonal and 2x + 6 the other.



Then we can create a right triangle with sides x and x + 3 and hypotenuse $\sqrt{89}$.
$x^2 + (x+3)^2 = \sqrt{89}^2$
$x^2 + x^2 + 6x + 9 = 89$
$2x^2 + 6x - 80 = 0$
$x^2 + 3x - 40 = 0$
$(x + 8) \times (x - 5) = 0$
$x = -8$; $x = 5$
x cannot be negative so $x = 5$.
Therefore, the diagonals $2x$ and 2x + 6 are 10 and 16, respectively.
The area of the rhombus is
$\frac{1}{2}d_1d_2$ where d_1 and d_2 are the 2 diagonals of the rhombus.
$\frac{1}{2} \times 10 \times 16 = 80$ **Ans.**

19. $(12 - x) \div 3x$ is a non-negative integer. So what is the largest possible integer value of x?
If x were negative, the numerator would always be positive (e.g., if $x = -3$, $12 - x$ would be $12 - (-3) = 12 + 3 = 15$). But the denominator would be negative leading to a negative value. If x is non-negative, then any value greater than 12 would give us a negative numerator but a positive denominator. So we need to consider only these values $0 \le x \le 12$.
12 is the largest value. What happens

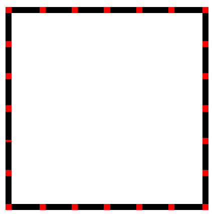
when $x = 12$?

$$\frac{12-12}{3\times 12} = \frac{0}{36} = 0$$

Notice, they did say **non-negative!**

12 **Ans.**

20. 24 4-inch wide square posts are evenly spaced 5 feet between adjacent posts to enclose a square field. So what is the outer perimeter of the field?



In the picture above, the red dots represent the 4-inch wide posts. The black lines represent the spacing between the posts If you look at any side of the square you will see 7 posts and 6 spaces.

$(7 \times \frac{1}{3}) + (6 \times 5) = \frac{7}{3} + 30 = 32\frac{1}{3}$

This is the side of the square. Thus, the perimeter of the square is

$4 \times 32\frac{1}{3} = 128\frac{4}{3} = 129\frac{1}{3}$ **Ans.**

21. What is the smallest solution of the equation $x^4 - 34x^2 + 225 = 0$?

$225 = 25 \times 9$ and $25 + 9 = 34$

Let's factor the equation.

$x^4 - 34x^2 + 225 = (x^2 - 9)(x^2 - 25) =$

$(x - 3)(x + 3)(x - 5)(x + 5) = 0$

$x = 3, -3, 5, -5$

The smallest solution is –5. **Ans.**

22. The arithmetic progressions {2, 5, 8, 11 …} and {3, 10, 17, 24 …} have some common values and we are asked to find the largest value less than 500 that they have in common.

The largest value of the first progression that is less than 500 is 497.

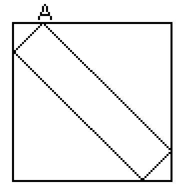
The largest value of the second progression that is less than 500 is 493.

497, 494, 491, 488, 485, 482, **479**

493, 486, **479**,

Bingo! 479 **Ans.**

23. An isosceles right triangle is removed from each corner of the square piece of paper shown below to create a rectangle. AB = 12 so what is the combined area of the four removed triangles?



Let w = the width of the rectangle.

Let l = the length of the rectangle.

If AB = 12, then $w^2 + l^2 = 12^2 = 144$

Now, let x = the side of the isosceles triangle whose hypotenuse is w.

Let y = the side of the isosceles triangle whose hypotenuse is l.

Then $2x^2 = w^2$ and $2y^2 = l^2$

The area of the triangle with sides x, x, and w is $\left(\frac{1}{2}x\right)(x) = \frac{1}{2}x^2$

There are two of these triangles so the total area of them is x^2.

The area of the triangle with sides y, y and l is $\left(\frac{1}{2}y\right)\times(y) = \frac{1}{2}y^2$

There are two of these triangles so the total area of them is y^2.

Thus the total area of the 4 triangles is $x^2 + y^2$.

But, remember that $2x^2 = w^2$ and $2y^2 = l^2$.

This means that

$x^2 = \frac{w^2}{2}$ and $y^2 = \frac{l^2}{2}$

Thus,

$x^2 + y^2 = \frac{w^2}{2} + \frac{l^2}{2} = \frac{1}{2}(w^2 + l^2) =$

$\frac{1}{2} \times 144 = 72$ **Ans.**

24. If $1 \le a \le 10$ and $1 \le b \le 36$, for how many ordered pairs of integers (a, b) is $\sqrt{a+\sqrt{b}}$ an integer?

First things first. $\sqrt{b}$ must resolve to an integer. For what values of b will this work?

1, 4, 9, 16, 25 and 36.

For these values, $\sqrt{b}$ resolves to 1, 2,

3, 4, 5, and 6, respectively.
Now the sum of *a* and one of these six values must be a perfect square so that the square root of the sum resolves to an integer.

If *a* = 1, then $\sqrt{b}$ would have to be 3, 8, or larger. 3 is good. **(1, 9)** is an ordered pair.

If *a* = 2, then $\sqrt{b}$ would have to be 2, 7 or larger. 2 is good. **(2, 4)** is an ordered pair.

If *a* = 3, then $\sqrt{b}$ would have to be 1, 6 or larger. 1 and 6 are both good. **(3, 1)** and **(3, 36)** are ordered pairs.

If *a* = 4, then $\sqrt{b}$ would have to be 5 or larger. 5 is good. **(4, 25)** is an ordered pair.

If *a* = 5, then $\sqrt{b}$ would have to be 4 or larger. 4 is good. **(5, 16)** is an ordered pair.

If *a* = 6, then $\sqrt{b}$ would have to be 3, 10 or larger. 3 is good. **(6, 9)** is an ordered pair.

If *a* = 7, then $\sqrt{b}$ would have to be 2, 9 or larger. 2 is good. **(7, 4)** is an ordered pair.

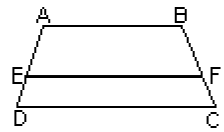
If *a* = 8, then $\sqrt{b}$ would have to be 1, 8 or larger. 1 is good. **(8, 1)** is an ordered pair.

If *a* = 9, then $\sqrt{b}$ would have to 7 or larger. There are no ordered pairs.

If a = 10, then $\sqrt{b}$ would have to be 6 or larger. 6 is good. **(10, 36)** is an ordered pair and we are done. We enumerated 10 ordered pairs. 10 **Ans.**

25. We are asked to find how many different combinations of 3 pairs of 6 students are possible.
Let's look at one example which always has 1 and 2 grouped together.
(1, 2), (3, 4), (5, 6)
The other combinations are
(1, 2), (3, 5), (4, 6)
(1, 2), (3, 6), (4, 5)
So when grouping (1,2) we have 3 combinations.
The same will hold true for (1, 3), (1, 4), (1,5) and (1,6).
So that is 3 × 5 = 15 combinations.
Are we done? Yes, because enumerating each of the others will cover all combinations. 15 **Ans.**

26. In trapezoid ABCD, AB || CD, AB = 7 and CD = 10. EF || AB and BF:FC = 3:4. So what is EF?



EF gets proportionately closer in size as it moves down the segments AD and BC.
10 – 7 = 3 which is the amount EF increases in size as it moves down AD and BC to become DC.

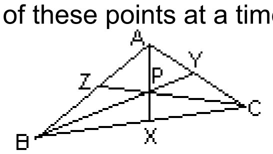
Since BF:FC = 3:4 BF = $\frac{3}{7}$ BC.

$$\frac{3}{7} \times 3 = \frac{9}{7}$$

$$\frac{9}{7} + 7 = \frac{9}{7} + \frac{49}{7} = \frac{58}{7}$$ **Ans.**

27. X is a multiple of 17, less than 1000 and one less than a multiple of 8. So what is the largest possible value of X?
X is one less than a multiple of 8 so X is a multiple of 17 that is odd.
The largest multiple of 17 less than 1000 is 986 which is even. 986 – 969 is odd. Is this 1 less than a multiple of 8? No. Subtract 34 from 969. That gives 935. Is this 1 less than a multiple of 8? Yes. 935 **Ans.**

28. Points X, Y and Z lie on the sides of triangle ABC so that the segments AX, BY and CZ intersect at point P. How many triangles can be formed by taking 3 of these points at a time?



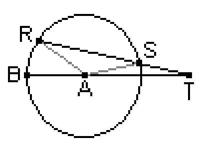
If 3 of these points at a time always made a triangle it there would be

$\frac{7!}{3!4!}$ = 35 triangles. But the catch is that not all of the combinations of 3 points can make a triangle. Here are the ones that can't:
AZB, BXC, AYC, APC, XPA and YPB.
All the rest can.

35 – 6 = 29 **Ans.**

29. The sum of *n* positive integers, not necessarily distinct, is 22. What is the largest possible product of the *n* integers?
Let's start with 11 × 11 = 121
Can we get this bigger?
10 × 10 × 2 = 200
9 × 9 × 4 = 324
8 × 8 × 6 = 384
7 × 7 × 7 ×1 = 343
6 × 6 × 6 × 4 = 864
5 × 5 × 5 × 5 × 2 = 1250
4 × 4 × 4 × 4 × 4 × 2 = 2048
3 × 3 × 3 × 3 × 3 × 3 × 3 × 1 = 2187
But there's a 1 in there. Combine 3 and 1.
3 × 3 × 3 × 3 × 3 × 3 × 4 = 2916
Finally 2^{11} = 2048.
I think that 2916 is as big as we can get.
2916 **Ans.**

30. In the figure, angle RAS is 74º and RTB is 28º. So what is the measure of minor arc BR?



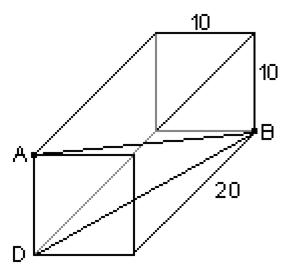
Triangle RAS is an isosceles triangle since AR and AS are radii of circle A. Therefore, the angle ASR and SRA are the same value.
Let *x* = the measure of angle SRA.
Then 2*x* + 74 = 180
2*x* = 106
x = 53
So angle SRA = angle ASR = 53º.
Angle TSA is supplementary to angle ASR. Therefore angle TSA =
180 – 53 = 127º.
We know that angle RTB is 28º.
Therefore, angle TAS =
180 – (28 + 127) = 180 – 155 = 25º.
This makes angle RAB =
180 – (25 + 74) =
180 – 99 = 81
The measure of a minor arc is the measure of its central angle.
81 **Ans.**

TARGET ROUND

1. It takes 6 small skeins of yarn to knit a scarf. We are asked to determine how many skeins it takes to make 15 scarves.
6 × 15 = 90 **Ans.**

2. Arithmetic sequence A starts with 30. Each term increases by 10.
Arithmetic sequence B starts with 30. Each term decreases by 10.
So what is the absolute value of the differences between the 51st term of sequence A and the 51st term of sequence B.
The 51st term of sequence A is
30 + (10 × 50) = 30 + 500 = 530
The 51st term of sequence B is
30 – (10 × 50) = 30 – 500 = –470
530 – (–470) = 530 + 470 = 1000 **Ans.**

3. We have a rectangular prism that measures 10 by 20 by 10. We are asked to find the length of the diagonal connecting point A and point B.



To find the length of diagonal AB, we must find the length of diagonal DB.
(We already know that AD = 10.)
Let *x* = DB.
$10^2 + 20^2 = x^2$
$100 + 400 = x^2$
$x^2 = 500$
We don't actually have to find x because we're going to use x^2 in the next equation.
Let *y* = AB. Then
$y^2 = 10^2 + x^2 = 100 + 500 = 600$
y = $\sqrt{600} = 10\sqrt{6}$ **Ans.**

4. We are asked to find the average daily high temperature in Addington from 9/15/08 to 9/19/08.
Just add up all the highs and divide by 5.
49 + 62 + 58 + 57 + 46 = 272

$\frac{272}{5} = 54.4$ **Ans.**

5. $\frac{7^{48} - 7^{47} - 7^{46}}{41} = 7^x$

$\frac{7^{46}\left(7^2 - 7 - 1\right)}{41} = 7^x$

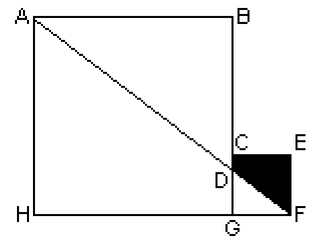
$\frac{7^{46}\left(41\right)}{41} = 7^x$

$7^{46} = 7^x$

$x = 46$ **Ans.**

6. Cindy wishes to arrange X coins into piles of Y coins each. There is at least 1 coin in each pile but a pile cannot have X coins. There are 13 possible values for Y so what is the smallest number of coins she could have?
What this question really boils down to is that we're looking for a number which has a certain number of divisors. But how many? Remember that we can't have a single pile, (i.e., no 1 × X) which means we're looking for 15 divisors including 1 and X. But when we normally create count up the divisors it's an even number (say divisors of 6 are 1 × 6 and 2 × 3 for 4 divisors). What this means is that X is a perfect square (so we have the square root of X times the square root of X). So what perfect square has 15 divisors?
Let's look at the first 15 or so perfect squares. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225
Any perfect squares that are created from prime numbers (like 3 × 3) would never have 15 divisors.
This leaves 16, 36, 64, 81, 100, 144, 196 and 225.
Any that are powers of a prime also won't get us there.
That leaves 36, 100, 144, 196, and 225.
The divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 24 and 36. No.
The divisors of 100 are 1, 2, 4, 5, 10, 20, 25, 50, 100 No.
The divisors of 144 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144.
That's 15 divisors. X = 144 **Ans.**

7. A 3-by 3 square adjoins a 10-by-10 square. We are asked to find the area of the shaded region.



To find the area of the shaded region we will first find the area of triangle DGF and subtract that from the area of the 3-by-3 square.
Triangle DGF is similar to triangle AHF. They share angle F in common. Angle DGF is 90º as is angle AHG which makes angle GDF and angle HAG the same.
We know that the length of segment GF is 3 and the length of segment HF is 13.
Let x = the length of DG.

$\frac{3}{13} = \frac{x}{10}$

$x = \frac{3 \times 10}{13} = \frac{30}{13}$

Therefore, the area of triangle DGF is

$\frac{1}{2} \times 3 \times \frac{30}{13} = \frac{45}{13}$

The area of the 3-by-3 square is 9.

$9 - \frac{45}{13} = \frac{117}{13} - \frac{45}{13} = \frac{72}{13}$ **Ans.**

8. The integers 1 through 16 is written on a separate slip of paper. Jillian draws slips from the pile without replacement until two of the numbers she has drawn from the pile have a product that is a perfect square. We are asked to find the maximum number of slips that Jillian can draw without obtaining a product that is a perfect square.
We need to start by figuring out which sets of 2 integers have a product that form a perfect square.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
The perfect squares must be less than 16 × 15 = 240
List them: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225
We can't make 1 because we don't have two 1's.
4 = 1 × 4
9 = 1 × 9
16 = 1 × 16, 2 × 8

25 has none
36 = 3 × 12, 4 × 9
49 has none
64 = 4 × 16
81 has none
100 has none
121 has none
144 = 9 × 16
169 has none
196 has none
225 has none
So let's list them again.
1 × 4, 1 × 9, 1 × 16, 2 × 8, 3 × 12, 4 × 9, 4 × 16, 9 × 16
This means that the "unsafe" numbers are 1, 2, 3, 4, 8, 9, 12 and 16.
The safe numbers are 5, 6, 7, 10, 11, 13, 14, and 15. That's 8 numbers. We must choose those first. Then what?
Since 2, 8, 3, and 12 appear only once (against one of the values in the list), we can choose any two of these as long as we don't choose any of the other 2.
So we're up to 10 numbers.
That leaves 1, 4, 9, and 16.
If we choose 1, we can't choose 4, 9 or 16.
If we choose 4, we can't choose 1, 9 or 16.
If we choose 9, we can't choose 1, 4 or 16.
If we choose 16, we can't choose 1, or 9, **but** we can choose 4.
Let's choose 16.

That does it. We have 11 numbers!
11 **Ans.**

TEAM ROUND

1. A rectangular box has dimensions 6x5x10. The box is filled with 3x3x3 cubes. So what percent of the volume of the box is taken up by the cubes?
Looking at the first level of boxes in the 6x5 dimension, we can only have two boxes. Those will take up 3 × 2 = 6 across. We can't have a second set of two boxes because in the length, there is only 5 – 3 = 2 inches left and we need 3. Well, how many levels of 2 boxes can we have? Only 3 levels because 3 × 3 = 9 and 10 – 9 = 1.
Therefore, we have 6 3x3x3 cubes. The volume of the cubes is
6 × 3 × 3 × 3 = 162
The volume of the 6x5x10 box is
6 × 5 × 10 = 300
$\frac{162}{300} = \frac{54}{100} =$ 54% **Ans.**

2. Planets X, Y and Z take 360, 450 and 540 days, respectively, to rotate around the same sun. The three planets are lined up in a ray with the sun as its endpoint. We are asked to find the minimum positive number of days before they are all in the exact same locations again.
This is just the LCM of 360, 450 and 540.
360 = 2 × 2 × 2 × 3 × 3 × 5
450 = 2 × 3 × 3 × 5 × 5
540 = 2 × 2 × 3 × 3 × 3 × 5
The LCM is:
2 × 2 × 2 × 3 × 3 × 3 × 5 × 5 =
8 × 27 × 25 = 5400 **Ans.**

3. You can only move vertically or horizontally to a square with a lesser fraction value. So what fraction is in the last square of your path? To make this easier, place the approximate decimal value in each entry of the maze.

9.67	3.33	3.27	3.17	4.5
4.2	3.5	3.6	3.05	3.1
4.75	4.78	2.75	2.88	2.9
4.77	4.5	2.71	3.09	2.5

We start at 4.75 in yellow. The second step is shown in pink. The third step is shown in green. The fourth step is colored in blue. The fifth step is colored in purple. The sixth step is colored in red. The seventh step in orange, the eight step in brown, the ninth step in olive and the tenth step in grey. (Whew! I was running outta colors here!!!) And the grey cell is 2.71, i.e., $\frac{19}{7}$. **Ans.**

4. The speed of a stream is 3 km/hr. A boat travels upstream at 12 km. It travels downstream to its original position along the same route. The speed of the boat in still water is 9 km/hr. We are asked to find the average speed of the boat for the entire round trip.
One way (whichever doesn't matter), the

boat is going with the stream. It takes advantage of the additional speed of the water so it actually goes 12 km/hr. It travels 12 km which takes 1 hour. Going the other way, the boat has to fight the stream so it only goes 9 – 3 = 6 km per hour. To go 12 km takes 2 hours. So, the boat travels 24 km in a total of 3 hr.

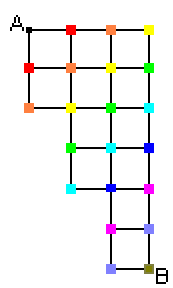
$\frac{24}{3} = 8$ **Ans.**

5. $x + \frac{1}{y} = 1$

$y + \frac{1}{z} = 1$

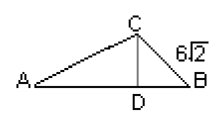
We are asked to find the value of *xyz*.
Multiply the first equation by *y*.
$xy + 1 = y$
Multiply the second equation by *z*.
$yz + 1 = z$; $yz = z - 1$
Multiply the third equation by *z*.
$xyz + z = yz$
$xyz + z = z - 1$
$xyz = -1$ **Ans.**

6. We have a ladybug walking on the segments of the diagram from point A to point B. The ladybug can move only to the right or downward and we must determine how many distinct paths are possible.



The diagrams shows the points colored to see the various steps that the ladybug can take. (red-orange-yellow-green-aqua-blue-purple-light blue-grey)
Let's look at the number of paths that exist starting at each of the yellow dots (the third step in the path).
The first yellow dot is reached by just going to the right from point A. There is just 1 path from there to point B.
The second yellow dot is reached by going right to the orange dot and then down, or going right to the red dot, down to the orange and then right, or going down to the red dot and then right to the orange and yellow dot. From this second yellow dot there are 6 paths to point B.
The third yellow dot can be reached by going right to the red dot and then down to the orange and yellow dot, or by going down to the red dot, right to the orange dot and then down to the yellow dot, or by going down to the red dot, down to the orange and right to the yellow dot. From there, there are 12 paths to point B.
So there is 1 way to get to the first yellow point, 3 ways to get to the second yellow point and 3 ways to get to the third yellow point.
(1 × 1) + (3 × 6) + (3 × 12) =
1 + 18 + 36 = 55 **Ans.**

7. Two angles of a triangle measure 30º and 45º. The side of the triangle opposite the 30º angle measures $6\sqrt{2}$ units and we must determine the sum of the lengths of the two remaining sides.



The triangle is drawn as triangle ABC. Angle A is 30º and Angle B is 45º. Drop a perpendicular from C to D and angle BCD must also be 45º. This makes triangle BCD an isosceles right triangle.
Let *x* = the length of BD.

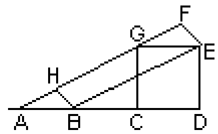
$x^2 + x^2 = \left(6\sqrt{2}\right)^2$

$2x^2 = 72$
$x^2 = 36$
$x = 6$
So the length of BD is 6 as is the length of CD.
Now, let's look at triangle ADC. In this case since angle A is 30º, angle DCA must be 60º. Segment AD is opposite angle DCA so it must be $6\sqrt{3}$ and the length of AC is 12.
The length of AB plus the length of AC is:
$6\sqrt{3} + 6 + 12 = 6\sqrt{3} + 18 \approx 28.3923 \approx$

28.4 **Ans.**

8. The mean of a set of 5 positive integers is 1.5 times the median. Three of the integers in the set are 24, 52 and 86. One of these values is the median. We are asked to determine the sum of all distinct possible sums of the other two integers.
Let x and y be the other 2 values.
If 24 is the median, we have
$x + y + 24 + 52 + 86 = x + y + 162$
$$\frac{x+y+162}{5} = 24 \times 1.5 = 36$$
$x + y + 162 = 180$
$x + y = 18$
Now suppose that 52 is the median.
$$\frac{x+y+162}{5} = 52 \times 1.5 = 78$$
$x + y + 162 = 390$
$x + y = 228$
Finally, suppose that 86 is the median.
$$\frac{x+y+162}{5} = 86 \times 1.5 = 129$$
$x + y + 162 = 645$
$x + y = 483$
18 + 228 + 483 = 729 **Ans.**

9. Quadrilateral CDEG is a square with CD = 3. Quadrilateral BEFH is a rectangle. BE = 5. We must find the length of BH.



Since BE is 5 and DE = CD = 3, BD must be 4 (i.e., 3, 4, 5 right triangle). Since CD = 3, BC must be 1. Triangle AHB is congruent to triangle GFE (angle F and angle H are 90º, HB = FE and angle FEG = angle HBA). Therefore, AB also is 3. This makes AC = 4 and since GC = 3, AHG must = 5. Finally, triangles AHB and BDE are similar. They each have a 90º angle and angle HBA = 90 – angle EBC = angle BED.
Let x = HB.
$$\frac{3}{5} = \frac{x}{3}$$
$$x = \frac{9}{5} = 1\frac{4}{5}$$ **Ans.**

10. We have a box with red, blue and green marbles. At least 17 marbles must be selected to make sure at least one of them is green. At least 18 marbles must be selected without replacement to be sure that at least 1 of them is red. And at least 20 marbles must be selected without replacement to be sure all three colors appear among the marbles selected. So how many marbles are in the box?
Let r = the number of red marbles.
Let b = the number of blue marbles.
Let g = the number of green marbles.
$r + b = 16$ (the 17th would be green)
$g + b = 17$ (the 18th would be red)
$r + g = 19$
$b = 16 - r$
$g + 16 - r = 17$
$g - r = 1$
$g + r = 19$
$2g = 20$
$g = 10$
$g - r = 1$
$10 - r = 1$
$r = 9$
$b = 16 - r$
$b = 16 - 9 = 7$
$r + b + g = 9 + 7 + 10 = 26$ **Ans.**